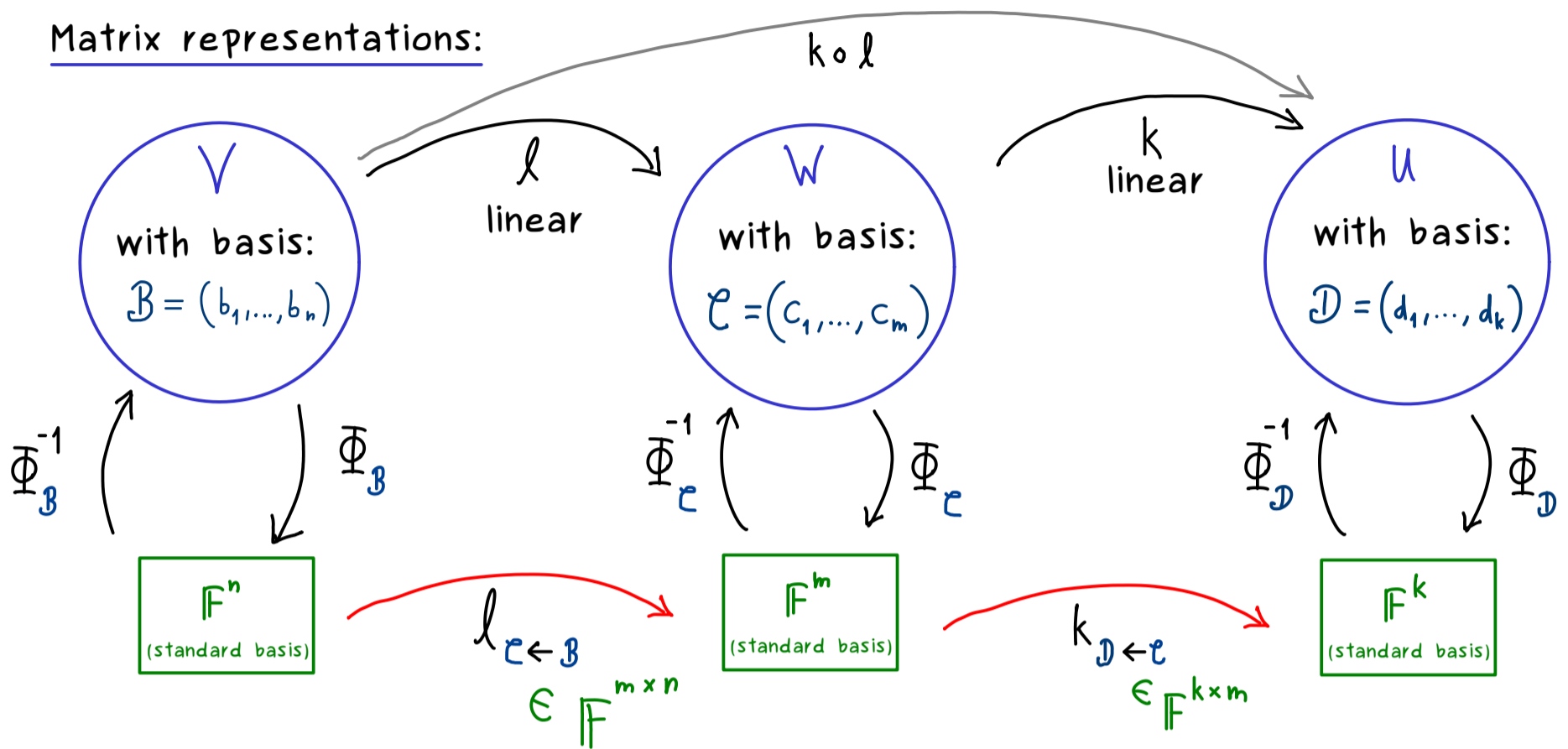


Abstract Linear Algebra - Part 26

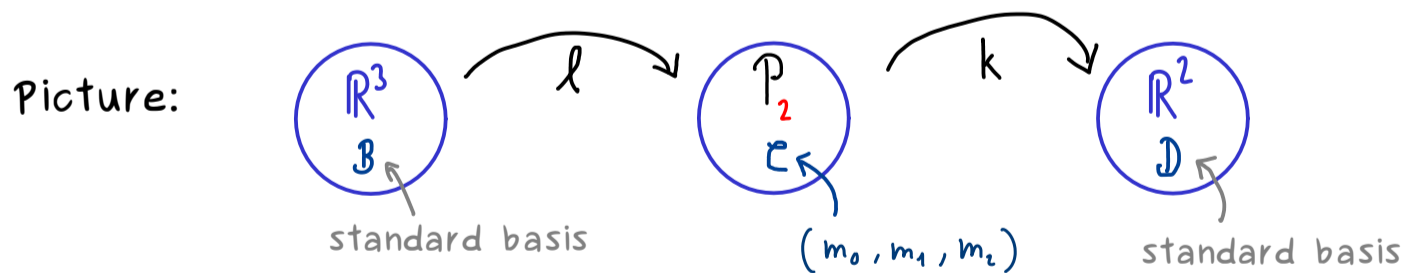
Matrix representations:



We get: $(k \circ l)_{\mathcal{D} \leftarrow \mathcal{B}} = k_{\mathcal{D} \leftarrow \mathcal{C}} l_{\mathcal{C} \leftarrow \mathcal{B}}$ (matrix product)

Example: $l: \mathbb{R}^3 \rightarrow \mathcal{P}_2(\mathbb{R})$, $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \mapsto (v_1 + v_2 + v_3) \cdot m_0 + (v_1 + v_2) \cdot m_1 + v_1 \cdot m_2$
 with $m_0: x \mapsto 1$, $m_k: x \mapsto x^k$

$k: \mathcal{P}_2(\mathbb{R}) \rightarrow \mathbb{R}^2$, $p \mapsto \begin{pmatrix} p'(1) \\ p(1) - p''(1) \end{pmatrix}$



$(k \circ l)_{\mathcal{D} \leftarrow \mathcal{B}} = ?$

$$l_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} | & | & | \\ \Phi_{\mathcal{C}}(l(b_1)) & \Phi_{\mathcal{C}}(l(b_2)) & \Phi_{\mathcal{C}}(l(b_3)) \\ | & | & | \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k_{\mathcal{D} \leftarrow \mathcal{C}} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$

$$(k \circ l)_{\mathcal{D} \leftarrow \mathcal{B}} = k_{\mathcal{D} \leftarrow \mathcal{C}} l_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

Corollary:

$$(l^{-1})_{\mathcal{B} \leftarrow \mathcal{C}} = (l_{\mathcal{C} \leftarrow \mathcal{B}})^{-1}$$

$n = m$

