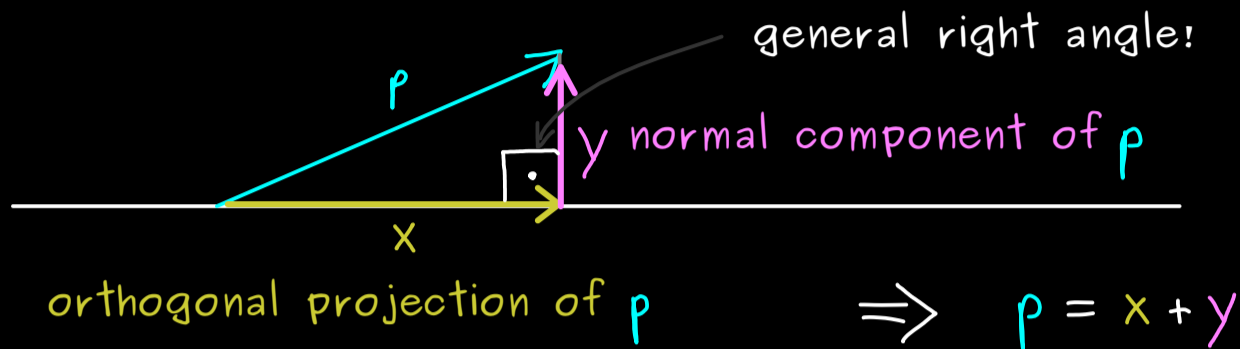


Abstract Linear Algebra - Part 13



Definition: V \mathbb{F} -vector space, inner product $\langle \cdot, \cdot \rangle : V \times V \longrightarrow \mathbb{F}$.

We say $x, y \in V$ are orthogonal, written as $x \perp y$,
if $\langle x, y \rangle = 0$.

Example: $\mathcal{P}([-1, 1], \mathbb{F})$ polynomial space, $\langle f, g \rangle = \int_{-1}^1 \overline{f(x)} g(x) dx$
 $p_1 : x \mapsto x$
 $p_2 : x \mapsto x^2 \Rightarrow \langle p_1, p_2 \rangle = \int_{-1}^1 x^3 dx = 0 \Rightarrow p_1 \perp p_2$

Definition: V \mathbb{F} -vector space, inner product $\langle \cdot, \cdot \rangle : V \times V \longrightarrow \mathbb{F}$.

For $M \subseteq V$, $M \neq \emptyset$, we define the orthogonal complement:

$$M^\perp := \left\{ x \in V \mid \langle x, m \rangle = 0 \text{ for all } m \in M \right\}$$

