

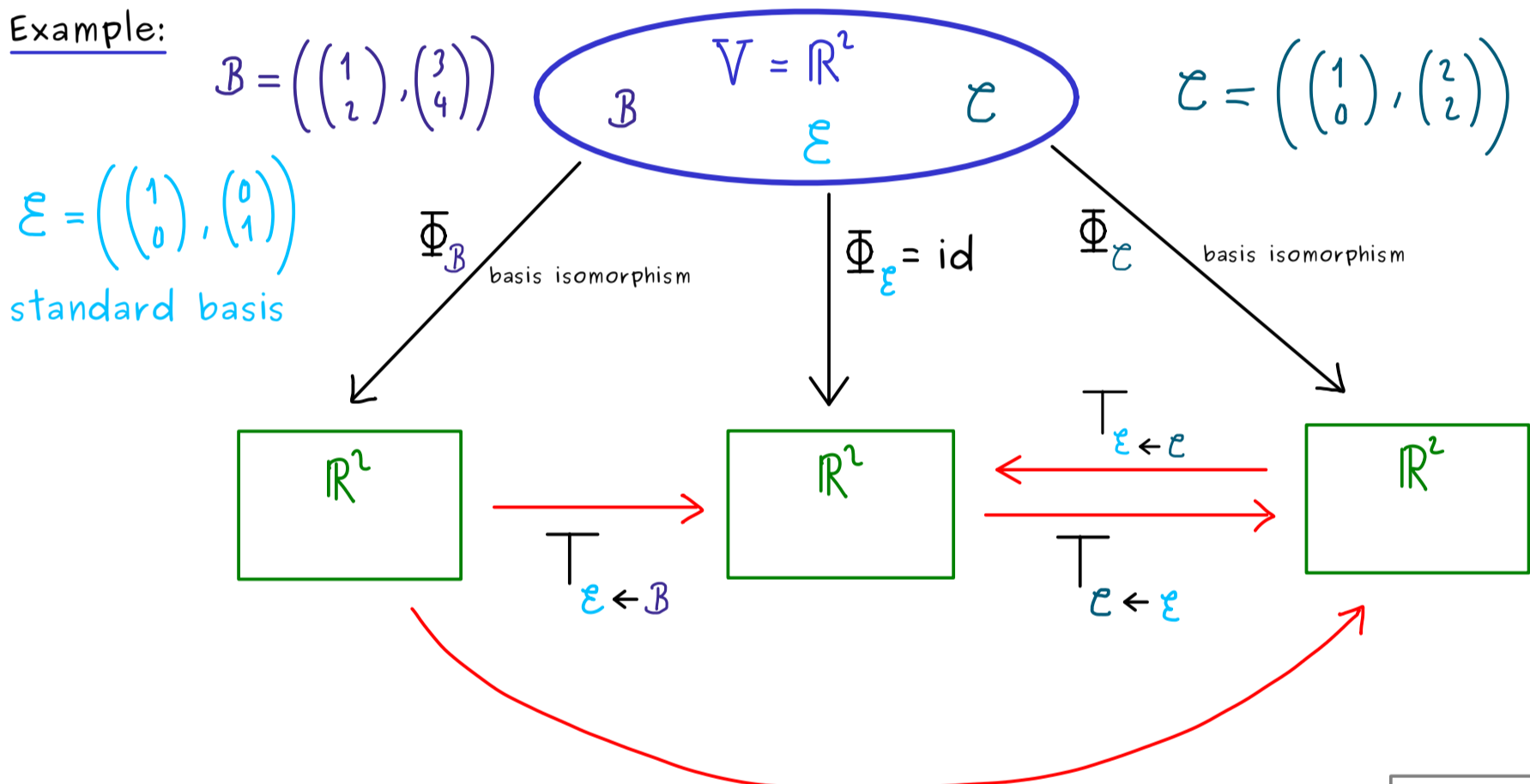
## Abstract Linear Algebra - Part 9

$$\underbrace{T_{\mathcal{B} \leftarrow \mathcal{C}}}_{\text{change-of-basis matrix}} \begin{pmatrix} \vdots \\ \vdots \end{pmatrix}_{\mathcal{C}} = \begin{pmatrix} \vdots \\ \vdots \end{pmatrix}_{\mathcal{B}}$$

$\Phi_{\mathcal{C}}(v)$        $\Phi_{\mathcal{B}}(v)$

$V$  vector space of dimension  $n$   
 $\Downarrow$   
 $T_{\mathcal{B} \leftarrow \mathcal{C}}$   $(n \times n)$ -matrix  
 invertible

Example:



We already know:

$$T_{\mathcal{E} \leftarrow \mathcal{B}} = \begin{pmatrix} | & | \\ \Phi_{\mathcal{E}}(b_1) & \Phi_{\mathcal{E}}(b_2) \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ b_1 & b_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$T_{\mathcal{E} \leftarrow \mathcal{C}} = \begin{pmatrix} | & | \\ \Phi_{\mathcal{E}}(c_1) & \Phi_{\mathcal{E}}(c_2) \\ | & | \end{pmatrix} = \begin{pmatrix} | & | \\ c_1 & c_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$$

$\mathcal{B} = \left( \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right)$

We can calculate:

$$\begin{aligned} T_{\mathcal{C} \leftarrow \mathcal{B}} &= T_{\mathcal{C} \leftarrow \mathcal{E}} T_{\mathcal{E} \leftarrow \mathcal{B}} \\ &= \underbrace{\left( T_{\mathcal{E} \leftarrow \mathcal{C}} \right)^{-1} T_{\mathcal{E} \leftarrow \mathcal{B}}}_{\text{calculate product immediately!}} \end{aligned}$$

$$\begin{aligned} \curvearrowright & T_{\mathcal{E} \leftarrow \mathcal{C}} X = T_{\mathcal{E} \leftarrow \mathcal{B}} \\ & \begin{matrix} \parallel & & \parallel \\ \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} & & \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \end{matrix} \end{aligned}$$

$$\Rightarrow \text{solve } \left( \begin{array}{cc|cc} 1 & 2 & 1 & 3 \\ 0 & 2 & 2 & 4 \end{array} \right) \xrightarrow{\text{II} \cdot \frac{1}{2}} \left( \begin{array}{cc|cc} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 \end{array} \right)$$

$$\xrightarrow{\text{I} - 2\text{II}} \left( \begin{array}{cc|cc} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 2 \end{array} \right)$$

$$X = T_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$$