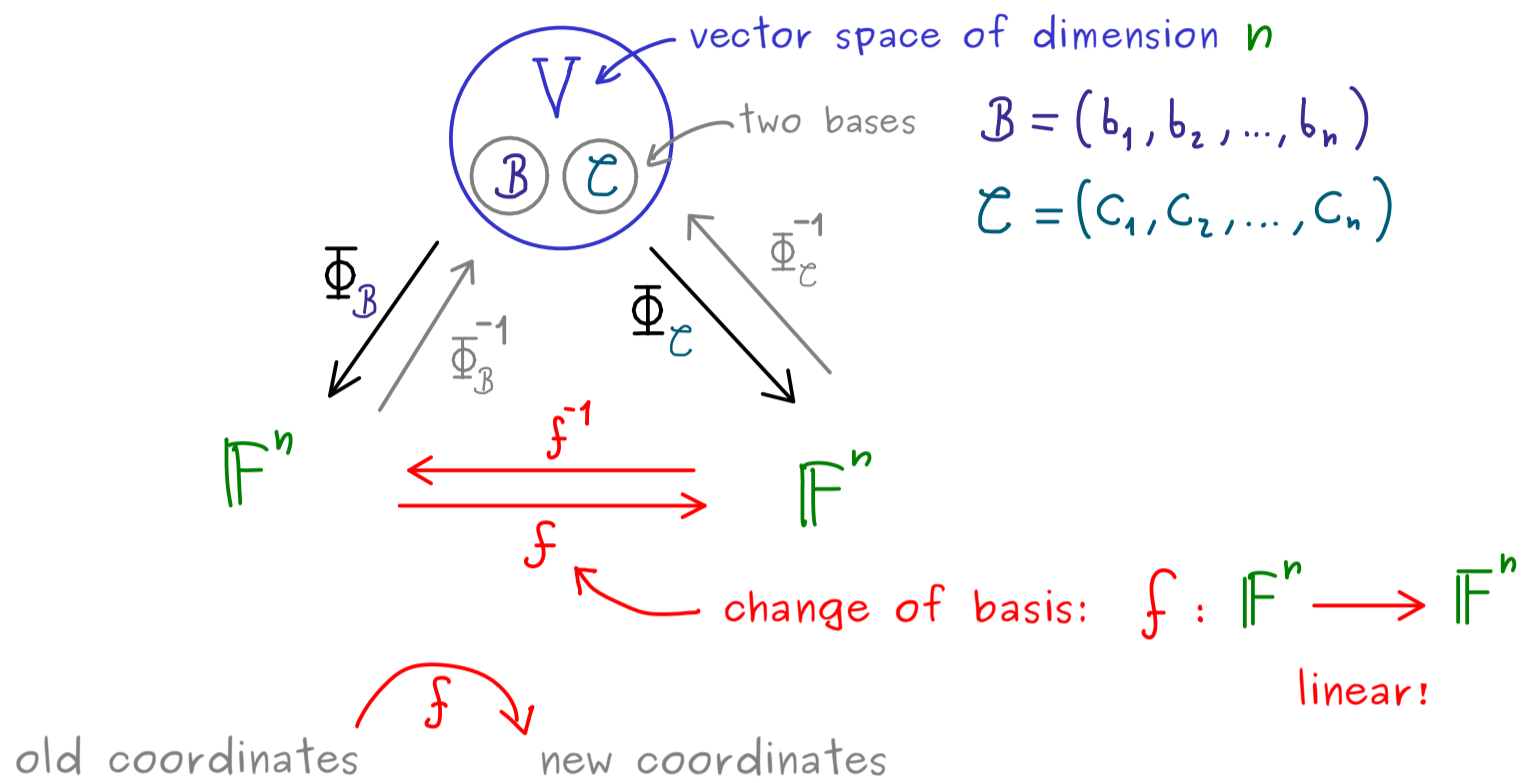


Abstract Linear Algebra - Part 8

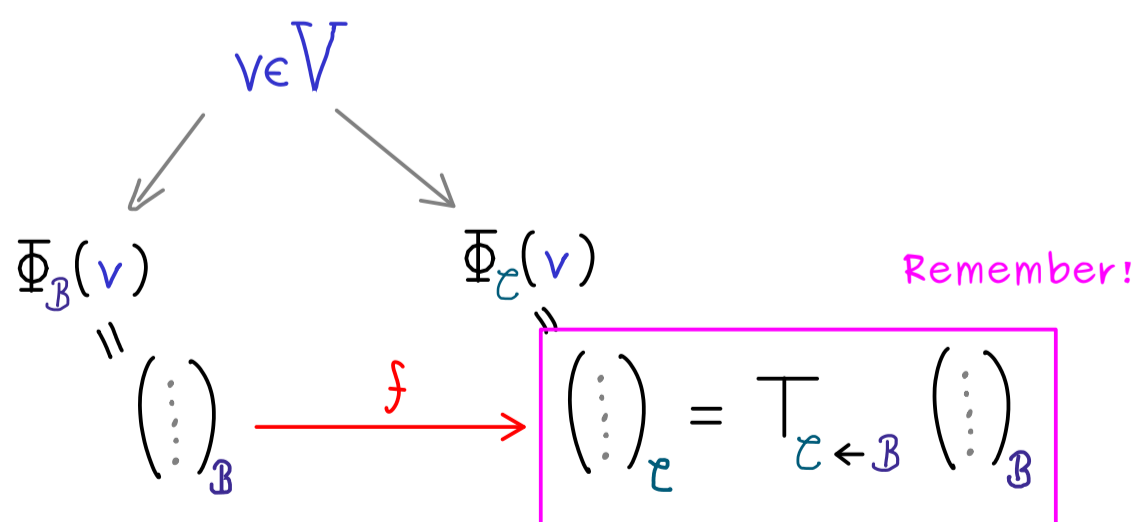


What happens if we put $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ into f ? $\leadsto f(e_1) = \Phi_C \left(\underbrace{\Phi_B^{-1}(e_1)}_{b_1} \right) = \Phi_C(b_1)$

We can see f as a matrix $f(x) = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix} x$

$$T_{C \leftarrow B} = \begin{pmatrix} | & | & & | \\ \Phi_C(b_1) & \Phi_C(b_2) & \dots & \Phi_C(b_n) \\ | & | & & | \end{pmatrix}$$

transformation matrix
transition matrix
change-of-basis matrix
from B to C



Fact: $\left(T_{C \leftarrow B} \right)^{-1} = T_{B \leftarrow C}$

Example: $V = \mathcal{P}_2(\mathbb{R})$ polynomials of degree ≤ 2

$$m_0: X \mapsto 1$$

$$m_1: X \mapsto X$$

$$m_2: X \mapsto X^2$$

$$\mathcal{B} = (\underbrace{m_2}_{b_1}, \underbrace{m_1}_{b_2}, \underbrace{m_0}_{b_3})$$

$$\mathcal{C} = (\underbrace{m_2 - \frac{1}{2}m_1}_{c_1}, \underbrace{m_2 + \frac{1}{2}m_1}_{c_2}, \underbrace{m_0}_{c_3})$$

$T_{\mathcal{C} \leftarrow \mathcal{B}}$ \rightsquigarrow how to write b_j with a linear combination of \mathcal{C}

$T_{\mathcal{B} \leftarrow \mathcal{C}}$ \rightsquigarrow how to write c_j with a linear combination of \mathcal{B}

\hookrightarrow column vectors $\Phi_{\mathcal{B}}(c_1) = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{pmatrix}$, $\Phi_{\mathcal{B}}(c_2) = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}$, $\Phi_{\mathcal{B}}(c_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$T_{\mathcal{B} \leftarrow \mathcal{C}} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{calculate inverse!}} T_{\mathcal{C} \leftarrow \mathcal{B}}$$