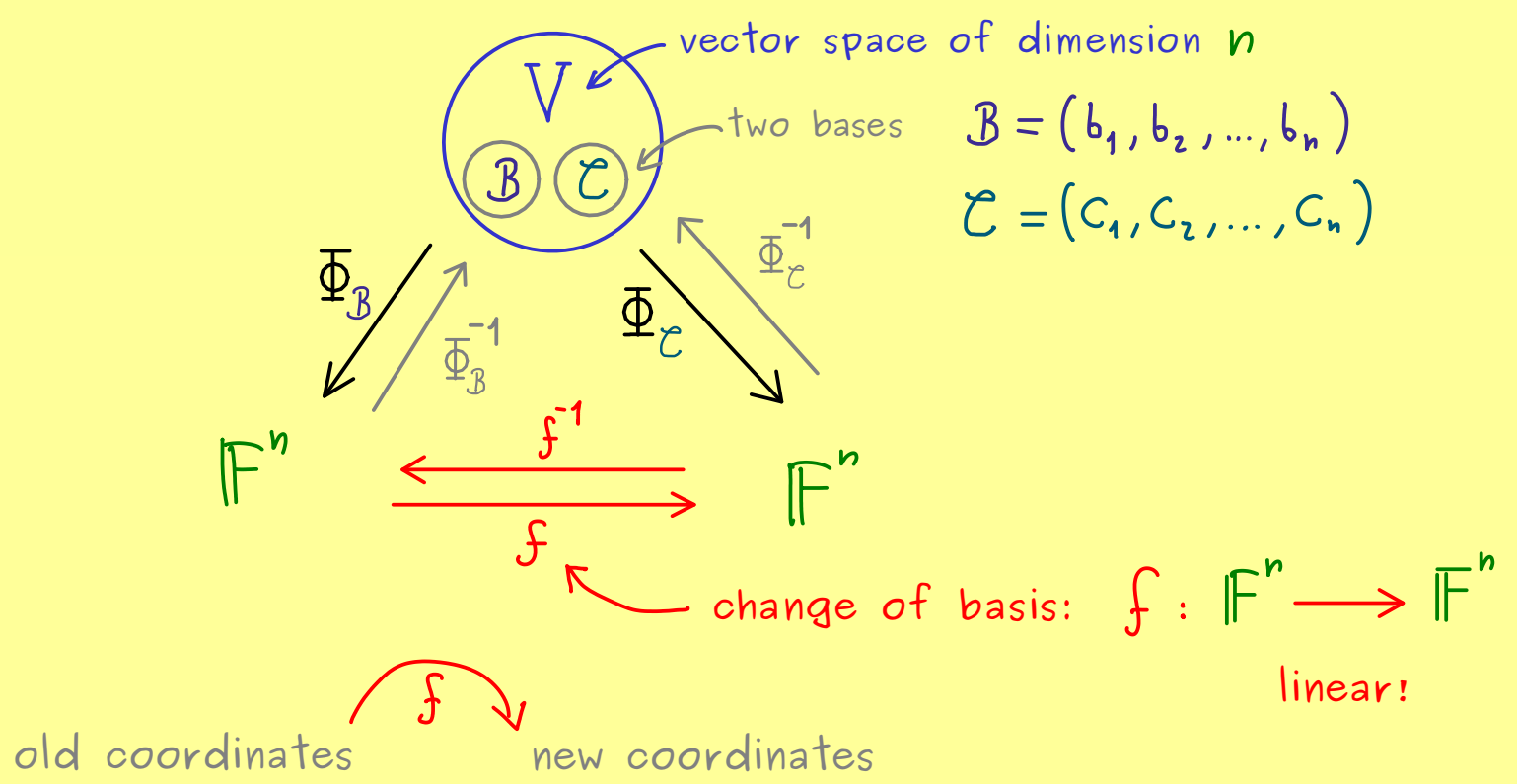




Abstract Linear Algebra - Part 8

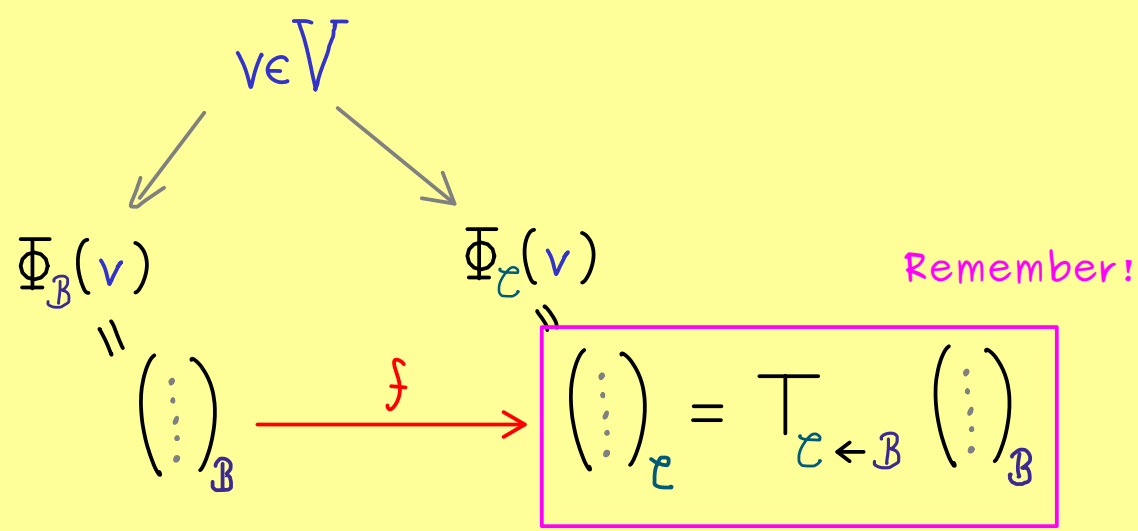


What happens if we put $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ into f ? $\rightsquigarrow f(e_1) = \Phi_C \left(\underbrace{\Phi_B^{-1}(e_1)}_{b_1} \right) = \Phi_C(b_1)$

We can see f as a matrix $f(x) = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \begin{pmatrix} | \\ x \\ | \end{pmatrix}$

$$T_{C \leftarrow B} = \begin{pmatrix} | & | & & | \\ \Phi_C(b_1) & \Phi_C(b_2) & \dots & \Phi_C(b_n) \\ | & | & & | \end{pmatrix}$$

transformation matrix
transition matrix
change-of-basis matrix
from B to C



Fact: $\left(T_{C \leftarrow B} \right)^{-1} = T_{B \leftarrow C}$

Example: $V = \mathcal{P}_2(\mathbb{R})$ polynomials of degree ≤ 2

- $m_0: x \mapsto 1$
- $m_1: x \mapsto x$
- $m_2: x \mapsto x^2$

$$B = \begin{pmatrix} \underbrace{m_1}_{b_1} & \underbrace{m_2}_{b_2} & \underbrace{m_0}_{b_3} \\ m_2 & m_1 & m_0 \end{pmatrix}$$

$$C = \begin{pmatrix} \underbrace{m_2 - \frac{1}{2}m_1}_{c_1} & \underbrace{m_2 + \frac{1}{2}m_1}_{c_2} & \underbrace{m_0}_{c_3} \\ m_2 & m_1 & m_0 \end{pmatrix}$$

$T_{C \leftarrow B} \rightsquigarrow$ how to write b_j with a linear combination of C

$T_{B \leftarrow C} \rightsquigarrow$ how to write c_j with a linear combination of B

\hookrightarrow column vectors $\Phi_B(c_1) = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{pmatrix}$, $\Phi_B(c_2) = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}$, $\Phi_B(c_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$T_{B \leftarrow C} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{calculate inverse!}} T_{C \leftarrow B}$$