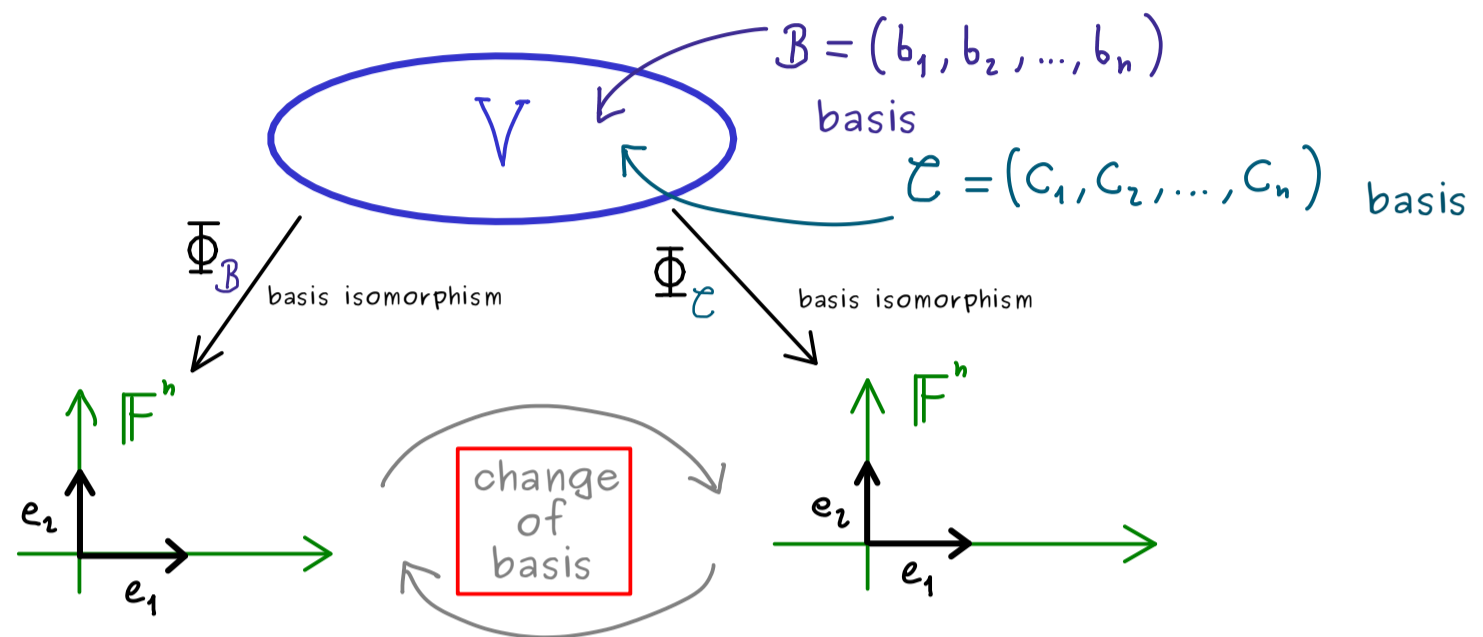


## Abstract Linear Algebra - Part 7



Recall:  $\Phi_B: V \rightarrow \mathbb{F}^n$  given by  $\Phi_B(b_j) = e_j$  for all  $j$

$\Phi_B^{-1}: \mathbb{F}^n \rightarrow V$  given by  $\Phi_B^{-1}(e_j) = b_j$  for all  $j$

For each  $v \in V$ :  $v = \Phi_B^{-1}\left(\begin{pmatrix} \text{coordinate} \\ \text{vector} \end{pmatrix}\right)$

Example:  $\mathcal{P}_2(\mathbb{R})$  with basis  $\mathcal{B} = (m_0, m_1, m_2)$  where  $m_0(x) = 1$ ,  $m_1(x) = x$ ,  $m_2(x) = x^2$

For  $p \in \mathcal{P}_2(\mathbb{R})$  given  $p(x) = 3x^2 + 8x - 2$

$$p = (-2) \cdot m_0 + 8 \cdot m_1 + 3 \cdot m_2 = \Phi_B^{-1}\left(\begin{pmatrix} -2 \\ 8 \\ 3 \end{pmatrix}\right) \quad \text{coordinate vector}$$

Another basis:  $\mathcal{C} = (c_1, c_2, c_3)$  with  $c_1 = m_0$ ,  $c_2 = m_1$ ,  $c_3$  polynomial

$$p = \Phi_C^{-1}\left(\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}\right) \quad \text{coordinate vector}$$

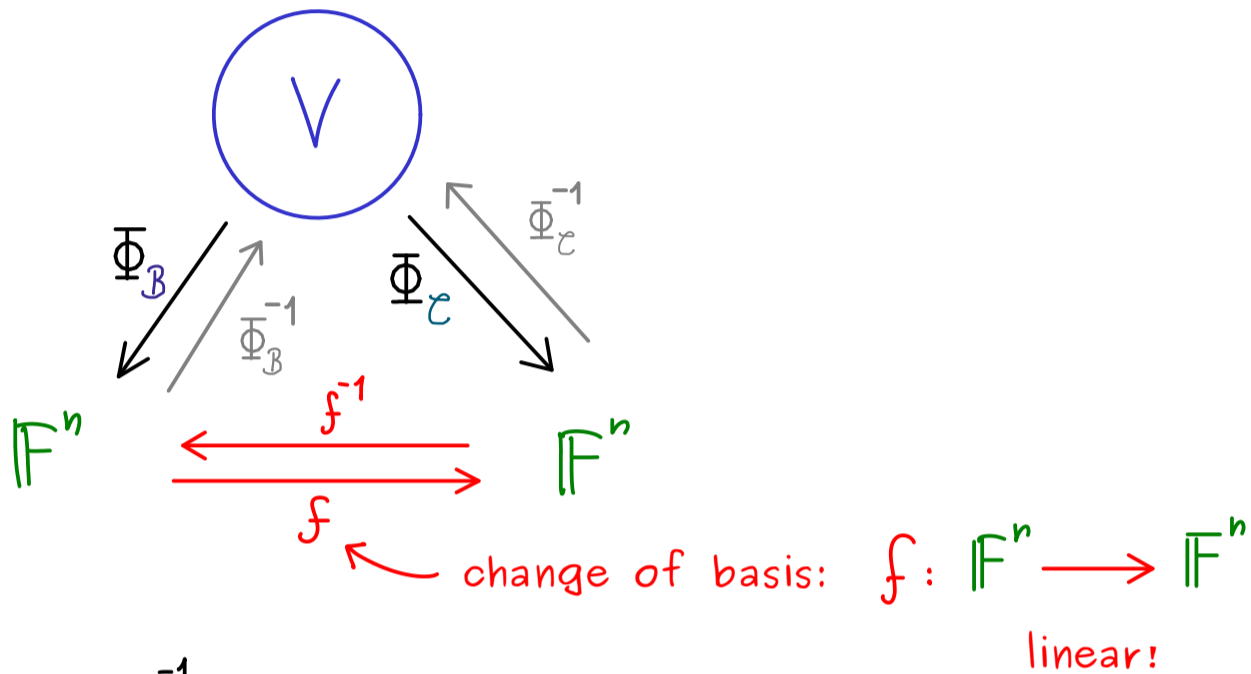
$\hookrightarrow c_3(x) = 3x^2 + 8x$

Old vs. new coordinates:  $\mathcal{B} = (b_1, b_2, \dots, b_n)$  basis,  $\mathcal{C} = (c_1, c_2, \dots, c_n)$  basis

$$\Phi_{\mathcal{B}}(v) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \longleftrightarrow \Phi_{\mathcal{C}}(v) = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{pmatrix}$$

$$v = \beta_1 b_1 + \dots + \beta_n b_n$$

$$v = \gamma_1 c_1 + \dots + \gamma_n c_n$$



We get:  $f(x) = \Phi_{\mathcal{C}} \circ \Phi_{\mathcal{B}}^{-1}(x)$