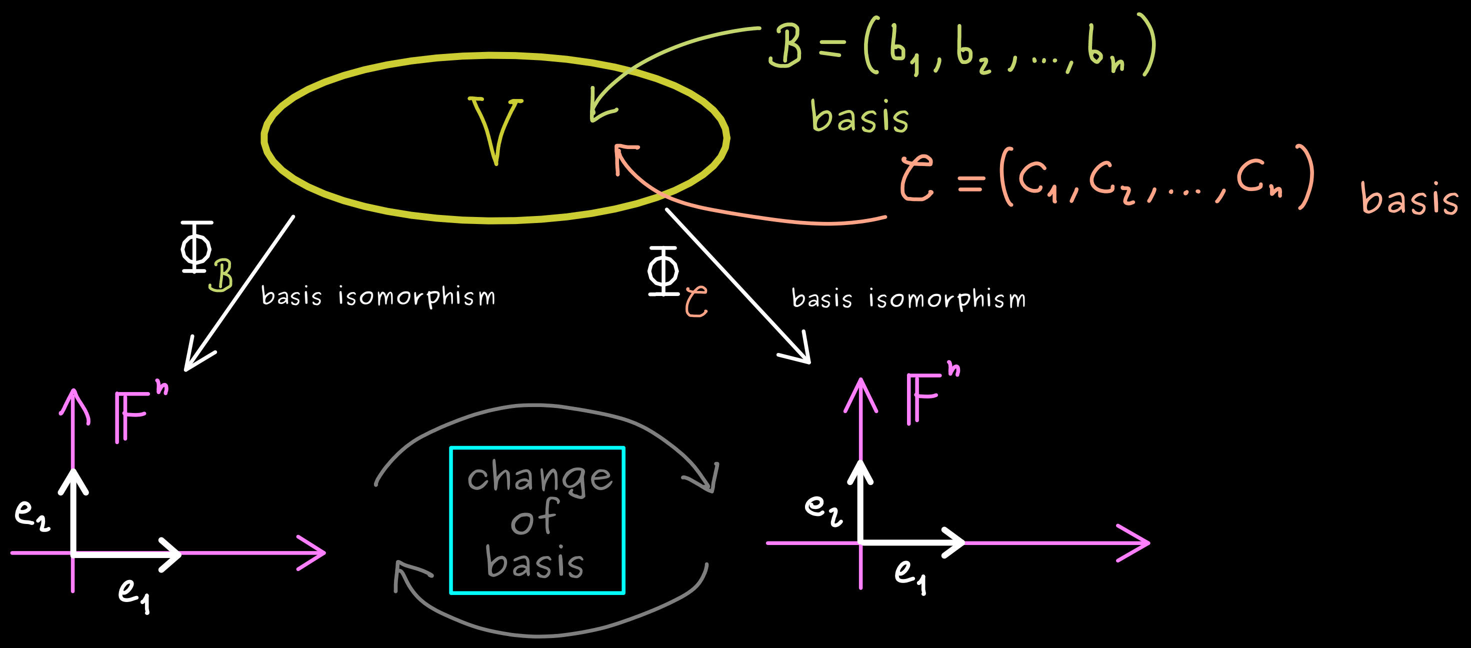
Abstract Linear Algebra - Part 7



Recall: $\Phi_{\mathcal{B}} : V \longrightarrow \mathbb{F}^n$ given by $\Phi_{\mathcal{B}}(b_j) = e_j$ for all j

$\Phi_{\mathcal{B}}^{-1} : \mathbb{F}^n \longrightarrow V$ given by $\Phi_{\mathcal{B}}^{-1}(e_j) = b_j$ for all j

For each $v \in V$: $\quad v = \Phi_{\mathcal{B}}^{-1}\big(\text{(coordinate vector)}\big)$

Example: $\mathcal{P}_2(\mathbb{R})$ with basis $\mathcal{B} = (m_0, m_1, m_2)$ where $m_0(x) = 1,\ m_1(x) = x,\ m_2(x) = x^2$

For $p \in \mathcal{P}_2(\mathbb{R})$ given $p(x) = 3x^2 + 8x - 2$

$$p = (-2)\cdot m_0 + 8 \cdot m_1 + 3 \cdot m_2 = \Phi_{\mathcal{B}}^{-1}\left(\begin{pmatrix} -2 \\ 8 \\ 3 \end{pmatrix}\right)$$

coordinate vector

Another basis: $\mathcal{C} = (c_1, c_2, c_3)$ with $c_1 = m_0$, $c_2 = m_1$, c_3 polynomial

$c_3(x) = 3x^2 + 8x$

$$p = \Phi_{\mathcal{C}}^{-1}\left(\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}\right)$$

coordinate vector

Old vs. new coordinates: $\mathcal{B} = (b_1, b_2, \dots, b_n)$ basis , $\mathcal{C} = (c_1, c_2, \dots, c_n)$ basis

$$\Phi_{\mathcal{B}}(v) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \quad \leftrightsquigarrow \quad \Phi_{\mathcal{C}}(v) = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{pmatrix}$$

$$v = \beta_1 \cdot b_1 + \cdots + \beta_n \cdot b_n \qquad\qquad v = \gamma_1 \cdot c_1 + \cdots + \gamma_n \cdot c_n$$

$$\begin{tikzcd}
 & V \arrow[dl, shift right, "\Phi_{\mathcal{B}}"'] \arrow[dr, shift left, "\Phi_{\mathcal{C}}"] & \\
\mathbb{F}^n \arrow[ur, shift right, "\Phi_{\mathcal{B}}^{-1}"'] \arrow[rr, shift right, "f"'] & & \mathbb{F}^n \arrow[ul, shift left, "\Phi_{\mathcal{C}}^{-1}"] \arrow[ll, shift right, "f^{-1}"']
\end{tikzcd}$$

change of basis: $f : \mathbb{F}^n \longrightarrow \mathbb{F}^n$

linear!

We get: $f(x) = \Phi_{\mathcal{C}} \circ \Phi_{\mathcal{B}}^{-1}(x)$