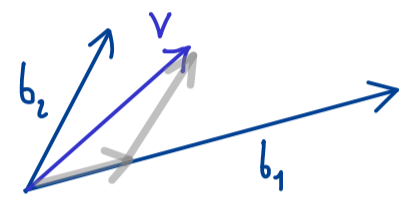


## Abstract Linear Algebra - Part 5

Coordinates with respect to a basis:

Assumptions:  $\mathbb{F} = \mathbb{R}$  or  $\mathbb{F} = \mathbb{C}$ ,  $V$   $\mathbb{F}$ -vector space with  $\dim(V) = n < \infty$ ,

$\mathcal{B} = (b_1, b_2, \dots, b_n)$  basis of  $V$ .



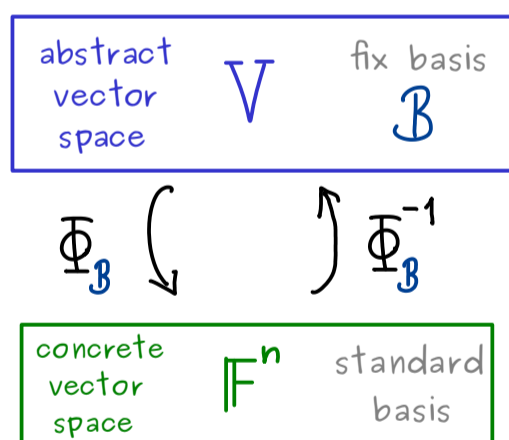
Then: each vector  $v \in V$  can be uniquely

written as:  $v = \alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_n b_n$  with  $\alpha_j \in \mathbb{F}$

Definition:  $\alpha_j$  are called the coordinates of  $v$  with respect to  $\mathcal{B}$ .

Remember:  $v = \sum_{j=1}^n \alpha_j b_j \iff \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \in \mathbb{F}^n$   
coordinate vector

Picture:



Define:  $\Phi_{\mathcal{B}}(\alpha_1 b_1 + \dots + \alpha_n b_n) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$

$\Phi_{\mathcal{B}} : V \longrightarrow \mathbb{F}^n$  is a linear map:

$$\Phi_{\mathcal{B}}(v+w) = \Phi_{\mathcal{B}}(v) + \Phi_{\mathcal{B}}(w)$$

$$\Phi_{\mathcal{B}}(\lambda \cdot v) = \lambda \cdot \Phi_{\mathcal{B}}(v)$$

$\Phi_{\mathcal{B}}$  is called basis isomorphism

$\hookrightarrow \Phi_{\mathcal{B}}(b_j) = e_j = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$  canonical unit vector