



Abstract Linear Algebra - Part 3

set + 8 rules

V \mathbb{F} -vector space

for example: space of functions

zero vector $0 \in V$

Question: $0 \cdot v = 0$ (zero in \mathbb{F}) \leftarrow zero vector, $(-1) \cdot v = -v$ for $v \in V$?

Proof:

$$0 \cdot v = (0 + 0) \cdot v \stackrel{(8)}{=} 0 \cdot v + 0 \cdot v$$

$$\stackrel{(3)}{\Rightarrow} 0 \cdot v + (-(0 \cdot v)) = 0 \cdot v + \underbrace{(0 \cdot v + (-(0 \cdot v)))}_{=0} \stackrel{\text{associativity (1)}}{=} 0 \cdot v$$

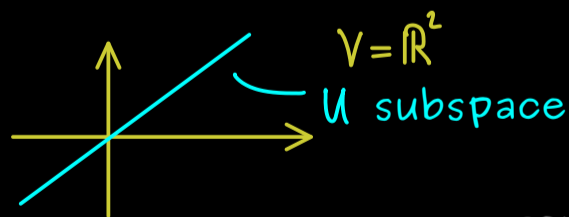
$$\stackrel{(3)}{\Rightarrow} 0 = 0 \cdot v$$

$$= (1 + (-1)) \cdot v \stackrel{(8)}{=} \underbrace{1 \cdot v}_{(6) \rightarrow v} + (-1) \cdot v$$

$$\stackrel{(3)}{\Rightarrow} -v + 0 = \underbrace{-v + v}_{=0} + (-1) \cdot v \Rightarrow -v = (-1) \cdot v \quad \checkmark$$

Linear subspace:

• vector space inside another one



• $\mathcal{P}(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R})$

zero function lies in $\mathcal{P}(\mathbb{R})$

adding two polynomials gives polynomial

scaling polynomial gives polynomial

Definition: V \mathbb{F} -vector space, $U \subseteq V$. If

(a) $0 \in U$,

(b) $u, v \in U \Rightarrow u + v \in U$,

(c) $u \in U, \lambda \in \mathbb{F} \Rightarrow \lambda \cdot u \in U$,

then U is also an \mathbb{F} -vector space. We call it a linear subspace of V .

Example: $\mathcal{P}_2(\mathbb{R})$ polynomials with degree ≤ 2 ($x \mapsto 4x^2 + x$, $x \mapsto 8x + 1$)

$\Rightarrow \mathcal{P}_2(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R})$ subspace