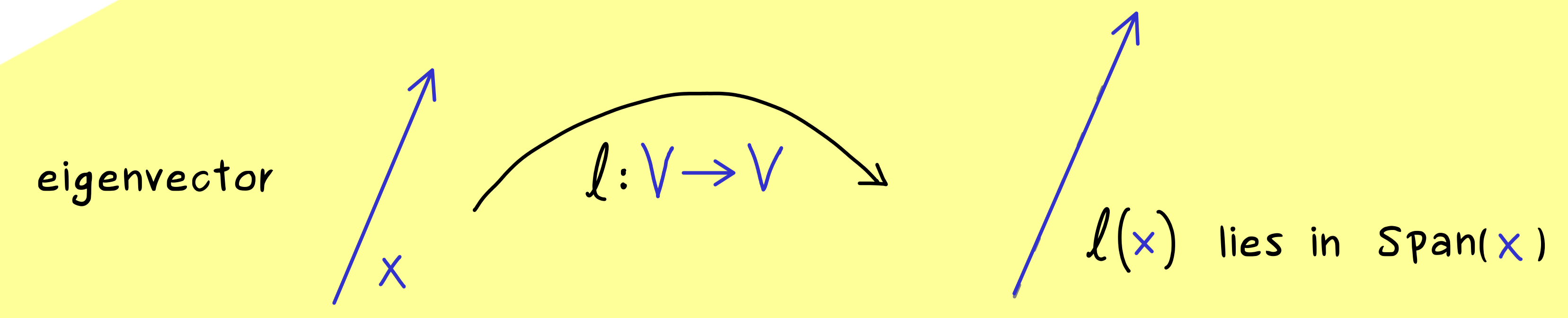
Abstract Linear Algebra - Part 34



Definition: Let V be an $\mathbb{F}$-vector space and $\ell : V \longrightarrow V$ be linear.

A vector $x \in V \setminus \{0\}$ is called an <u>eigenvector of ℓ</u> if

$$\exists \lambda \in \mathbb{F} \; : \quad \ell(x) = \lambda \cdot x$$

<u>eigenvalue of ℓ</u> associated to x

Remember: If $\mathrm{Ker}(\ell - \lambda \cdot \mathrm{id}) \neq \{0\}$, then λ is an eigenvalue of ℓ.

$\underbrace{\mathrm{Ker}(\ell - \lambda \cdot \mathrm{id})}\setminus\{0\}$ <u>set of all eigenvectors</u> of ℓ associated to λ

<u>eigenspace</u> of ℓ associated to λ

For the finite dimensional case: Let $\mathcal{B}$ be a basis V.

Then: $(\ell - \lambda \cdot \mathrm{id})_{\mathcal{B} \leftarrow \mathcal{B}} = \ell_{\mathcal{B} \leftarrow \mathcal{B}} - \lambda \cdot \mathbb{1}$

Hence: $\mathrm{Ker}(\ell - \lambda \cdot \mathrm{id}) \neq \{0\} \iff \mathrm{Ker}(\ell_{\mathcal{B} \leftarrow \mathcal{B}} - \lambda \cdot \mathbb{1}) \neq \{0\}$

λ eigenvalue of $\ell \iff \lambda$ eigenvalue of $\ell_{\mathcal{B} \leftarrow \mathcal{B}}$

$\det(\ell - \lambda \cdot \mathrm{id}) = 0 \iff \det(\ell_{\mathcal{B} \leftarrow \mathcal{B}} - \lambda \cdot \mathbb{1}) = 0$

Example: $V = C^{\infty}(\mathbb{R}) := \left\{ f : \mathbb{R} \rightarrow \mathbb{R} \;\middle|\; f \text{ is arbitrarily often continuously differentiable} \right\}$

$\ell : V \longrightarrow V \, , \quad f \mapsto f'$ linear map

$\boxed{\exp: \; x \mapsto e^{x}}$

$\ell(\exp) = \exp$

eigenvalue: 1

eigenvector = <u>eigenfunction</u>