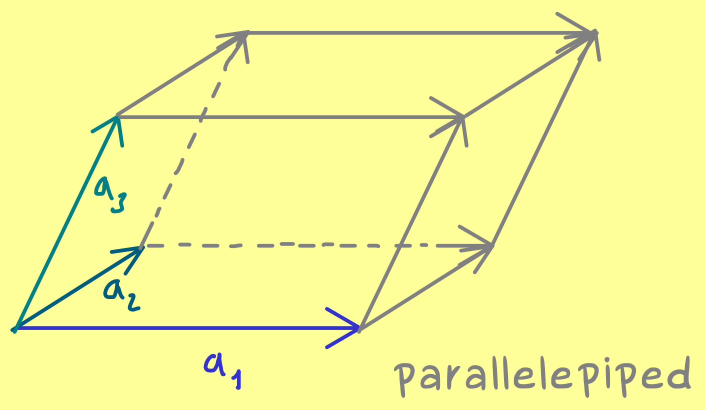
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Abstract Linear Algebra - Part 33

Determinant function: $\det : \underbrace{\mathbb{R}^n \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{n \text{ times}} \longrightarrow \mathbb{R}$



$\rightsquigarrow \det(A) \in \mathbb{C}$ for $A \in \mathbb{C}^{n \times n}$

For a linear map $f_A : \mathbb{C}^n \longrightarrow \mathbb{C}^n$, $x \mapsto Ax$

represented by $A \in \mathbb{C}^{n\times n}$, define:

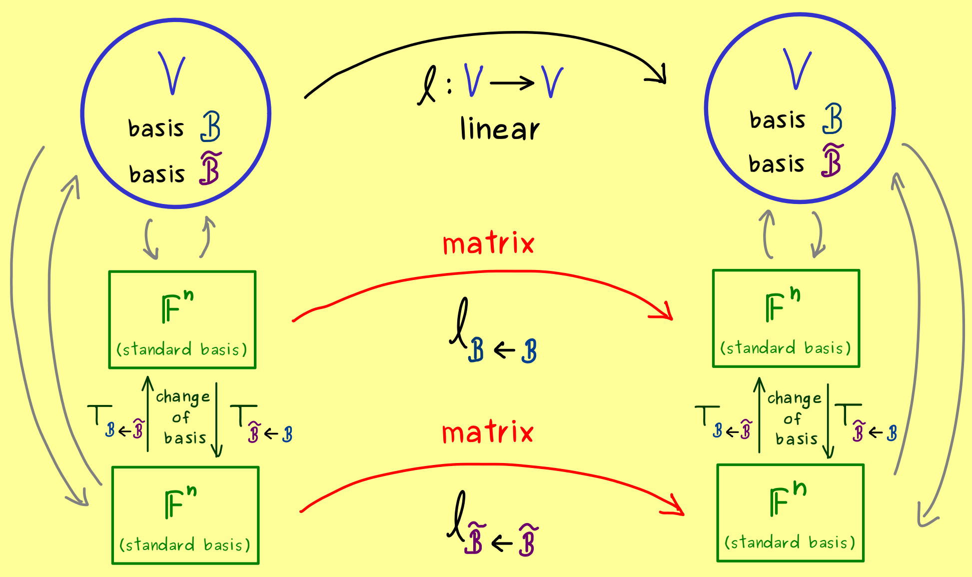
$$\det(f_A) := \det(A)$$

Definition: V $\mathbb{F}$-vector space, $\ell : V \longrightarrow V$ linear, $\mathcal{B} = (b_1, \dots, b_n)$ basis of V.

Then the matrix representation $\ell_{\mathcal{B} \leftarrow \mathcal{B}} \in \mathbb{F}^{n \times n}$ is a square matrix

and we can set: $\det(\ell) := \det(\ell_{\mathcal{B}\leftarrow\mathcal{B}})$.

What happens if we take another basis $\tilde{\mathcal{B}}$?



$$\ell_{\tilde{\mathcal{B}}\leftarrow\tilde{\mathcal{B}}} = T_{\tilde{\mathcal{B}}\leftarrow\mathcal{B}} \, \ell_{\mathcal{B}\leftarrow\mathcal{B}} \left(T_{\tilde{\mathcal{B}}\leftarrow\mathcal{B}}\right)^{-1}$$

similar matrices

$$\det(TAT^{-1}) = \det(T)\det(A)\det(T)^{-1}$$
$$= \det(A)$$

$$\Longrightarrow \det(\ell_{\tilde{\mathcal{B}}\leftarrow\tilde{\mathcal{B}}}) = \det(\ell_{\mathcal{B}\leftarrow\mathcal{B}})$$

Properties: $\det(\ell \circ k) = \det(\ell) \cdot \det(k)$

$\det(id) = 1$

$\det(\ell^{-1}) = \det(\ell)^{-1}$