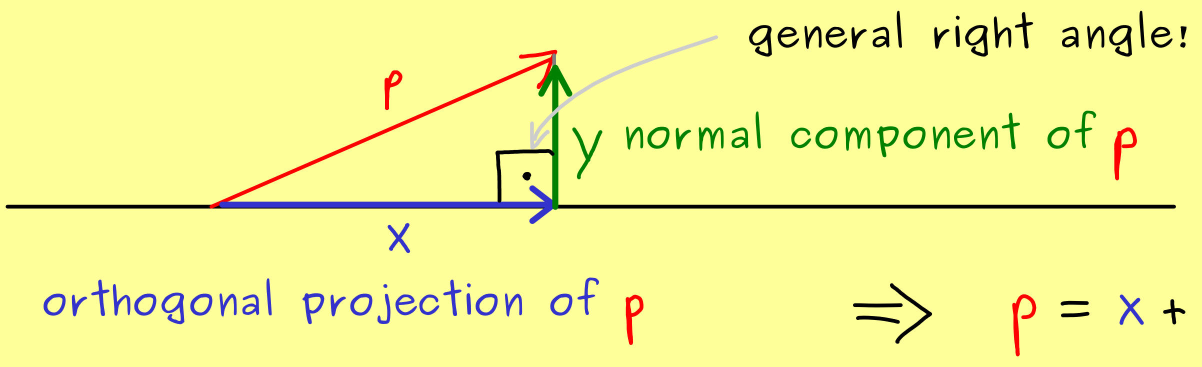
Abstract Linear Algebra – Part 13



$\Rightarrow \quad p = x + y$

Definition: V $\mathbb{F}$-vector space, inner product $\langle \cdot , \cdot \rangle : V \times V \longrightarrow \mathbb{F}$.

We say $x, y \in V$ are orthogonal, written as $x \perp y$,

if $\langle x, y \rangle = 0$.

Example: $\mathcal{P}([-1,1], \mathbb{F})$ polynomial space, $\langle f, g \rangle = \int_{-1}^{1} \overline{f(x)}\, g(x)\, dx$

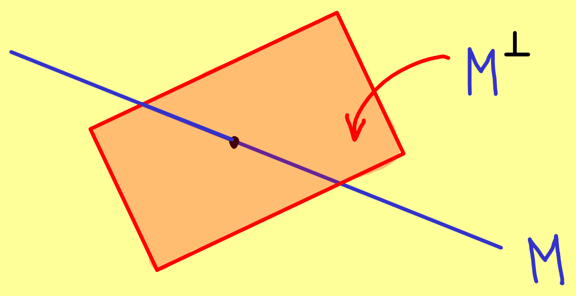
$p_1 : x \mapsto x$, $p_2 : x \mapsto x^2$ $\Rightarrow \langle p_1, p_2 \rangle = \int_{-1}^{1} x^3\, dx = 0 \Rightarrow p_1 \perp p_2$

Definition: V $\mathbb{F}$-vector space, inner product $\langle \cdot , \cdot \rangle : V \times V \longrightarrow \mathbb{F}$.

For $M \subseteq V$, $M \neq \emptyset$, we define the orthogonal complement:

$$M^{\perp} := \left\{ x \in V \;\middle|\; \langle x, m \rangle = 0 \text{ for all } m \in M \right\}$$

in $\mathbb{R}^3$:



in $\mathbb{R}^2$: