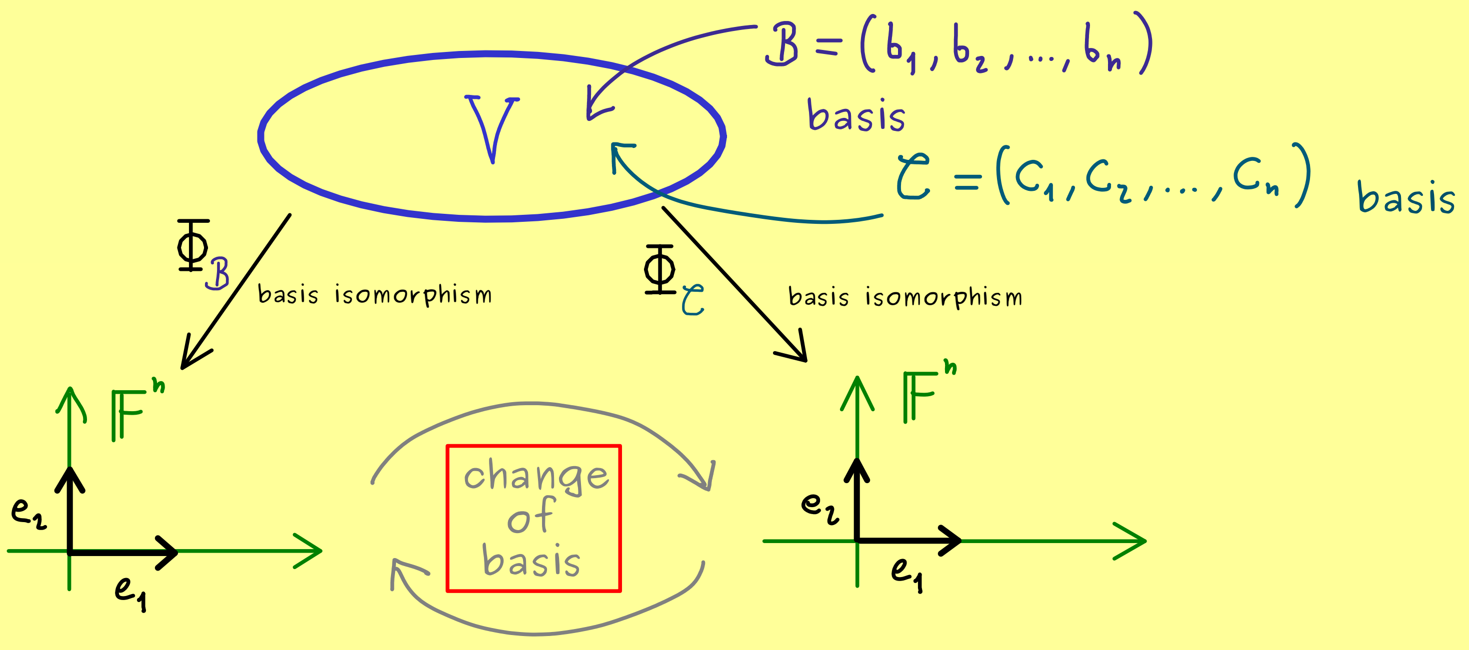
BECOME A MEMBER
ON STEADY

The Bright Side of Mathematics

Abstract Linear Algebra – Part 7



<u>Recall:</u> $\Phi_\mathcal{B} : V \longrightarrow \mathbb{F}^n$ given by $\Phi_\mathcal{B}(b_j) = e_j$ for all j

$\Phi_\mathcal{B}^{-1} : \mathbb{F}^n \longrightarrow V$ given by $\Phi_\mathcal{B}^{-1}(e_j) = b_j$ for all j

For each $v \in V$: $\quad v = \Phi_\mathcal{B}^{-1}\left(\begin{pmatrix} \text{coordinate} \\ \text{vector} \end{pmatrix} \right)$

<u>Example:</u> $\mathcal{P}_2(\mathbb{R})$ with basis $\mathcal{B} = (m_0, m_1, m_2)$ where $m_0(x) = 1,\ m_1(x) = x,\ m_2(x) = x^2$

For $p \in \mathcal{P}_2(\mathbb{R})$ given $p(x) = 3x^2 + 8x - 2$

$$p = (-2)\cdot m_0 + 8 \cdot m_1 + 3 \cdot m_2 = \Phi_\mathcal{B}^{-1}\left(\begin{pmatrix} -2 \\ 8 \\ 3 \end{pmatrix} \right)$$

coordinate vector

Another basis: $\mathcal{C} = (c_1, c_2, c_3)$ with $c_1 = m_0$, $c_2 = m_1$, c_3 polynomial

$c_3(x) = 3x^2 + 8x$

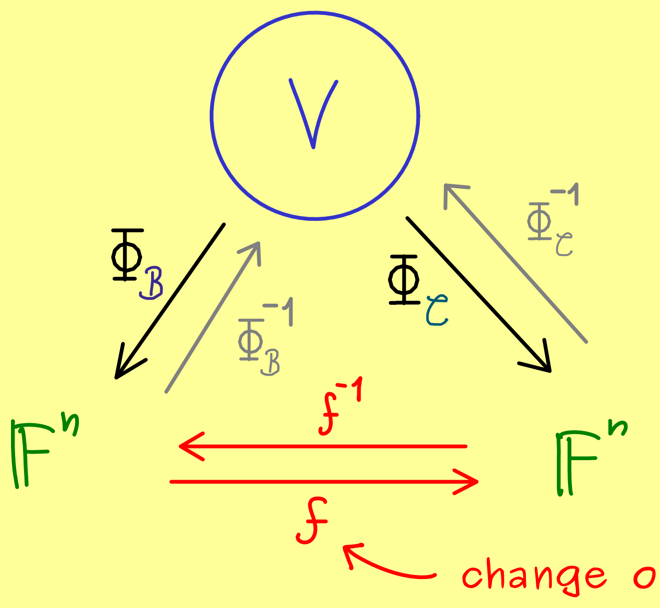
$$p = \Phi_\mathcal{C}^{-1}\left(\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right)$$

coordinate vector

<u>Old vs. new coordinates:</u> $\mathcal{B} = (b_1, b_2, \dots, b_n)$ basis , $\mathcal{C} = (c_1, c_2, \dots, c_n)$ basis

$$\Phi_\mathcal{B}(v) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \quad \longleftrightarrow \quad \Phi_\mathcal{C}(v) = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{pmatrix}$$

$v = \beta_1 \cdot b_1 + \dots + \beta_n \cdot b_n \qquad v = \gamma_1 \cdot c_1 + \dots + \gamma_n \cdot c_n$



change of basis: $f : \mathbb{F}^n \longrightarrow \mathbb{F}^n$

linear!

We get: $f(x) = \Phi_\mathcal{C} \circ \Phi_\mathcal{B}^{-1}(x)$