BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Abstract Linear Algebra – Part 3

$\boxed{\text{set + 8 rules}}$ V $\mathbb{F}$-vector space

for example: space of functions

zero vector $o \in V$

Question: $0 \cdot v = o$ ← zero vector , $(-1)\cdot v = -v$ for $v \in V$?

↑ zero in $\mathbb{F}$

Proof: $0\cdot v = (0+0)\cdot v \overset{(8)}{=} 0\cdot v + 0\cdot v$ associativity (1)

$$\overset{(3)}{\Longrightarrow} 0\cdot v + (-(0\cdot v)) = 0\cdot v + \underbrace{\big(0\cdot v + (-(0\cdot v))\big)}_{=o}$$

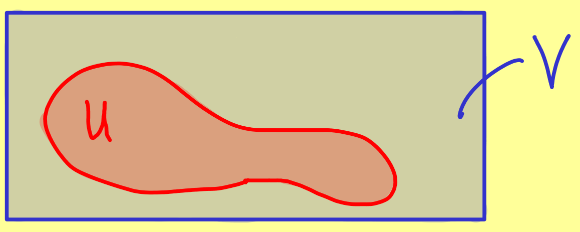
$$\overset{(3)}{\Longrightarrow} o = 0\cdot v \quad \checkmark$$

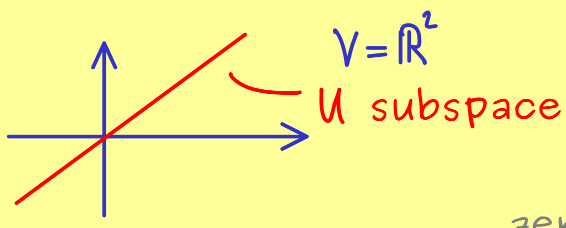
$$= (1 + (-1))\cdot v \overset{(8)}{=} \underbrace{1\cdot v}_{(6)\nearrow v} + (-1)\cdot v$$

$$\overset{(3)}{\Longrightarrow} -v + o = \underbrace{-v + v}_{=o} + (-1)\cdot v \Longrightarrow -v = (-1)\cdot v \quad \checkmark$$

Linear subspace:

- vector space inside another one



- 

- $\mathcal{P}(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R})$
  - zero function lies in $\mathcal{P}(\mathbb{R})$
  - adding two polynomials gives polynomial
  - scaling polynomial gives polynomial

Definition: V $\mathbb{F}$-vector space, $U \subseteq V$. If

(a) $o \in U$,

(b) $u, v \in U \Longrightarrow u + v \in U$,

(c) $u \in U, \lambda \in \mathbb{F} \Longrightarrow \lambda \cdot u \in U$,

then U is also an $\mathbb{F}$-vector space. We call it a linear subspace of V.

Example: $\mathcal{P}_2(\mathbb{R})$ polynomials with degree ≤ 2 $\quad (x \mapsto 4x^2 + x \;,\; x \mapsto 8x + 1)$

$$\Longrightarrow \mathcal{P}_2(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R}) \quad \text{subspace}$$