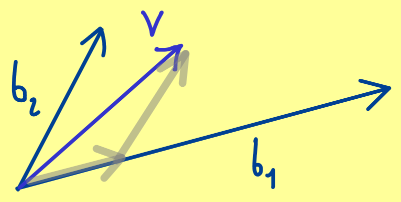
# Abstract Linear Algebra – Part 5

## Coordinates with respect to a basis:

Assumptions: $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$, $V$ $\mathbb{F}$-vector space with $\dim(V) = n < \infty$,

$\mathcal{B} = (b_1, b_2, \dots, b_n)$ basis of $V$.

Then: each vector $v \in V$ can be uniquely written as:




$$v = \alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_n b_n \quad \text{with} \quad \alpha_j \in \mathbb{F}$$

Definition: $\alpha_j$ are called the coordinates of $v$ with respect to $\mathcal{B}$.

Remember: $v = \sum_{j=1}^{n} \alpha_j b_j \quad \xleftrightarrow{1:1} \quad \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \in \mathbb{F}^n$

coordinate vector

Picture:

$$\begin{array}{c} \boxed{\text{abstract vector space} \quad V \quad \text{fix basis } \mathcal{B}} \\ \Phi_{\mathcal{B}} \downarrow \quad \uparrow \Phi_{\mathcal{B}}^{-1} \\ \boxed{\text{concrete vector space} \quad \mathbb{F}^n \quad \text{standard basis}} \end{array}$$

Define: $\Phi_{\mathcal{B}}(\alpha_1 b_1 + \dots + \alpha_n b_n) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$

$\Phi_{\mathcal{B}} : V \longrightarrow \mathbb{F}^n$ is a linear map:

$$\Phi_{\mathcal{B}}(v + w) = \Phi_{\mathcal{B}}(v) + \Phi_{\mathcal{B}}(w)$$

$$\Phi_{\mathcal{B}}(\lambda \cdot v) = \lambda \cdot \Phi_{\mathcal{B}}(v)$$

$\Phi_{\mathcal{B}}$ is called basis isomorphism

$$\Phi_{\mathcal{B}}(b_j) = e_j = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{canonical unit vector}$$