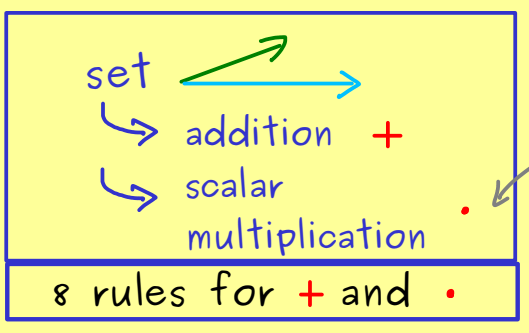
The Bright Side of
Mathematics

Abstract Linear Algebra – Part 2

vector space:



Examples: (a) The space of matrices $\mathbb{C}^{m\times n}$ with matrix addition and scaling:

complex vector space (see: Linear Algebra – Part 11 and 58)

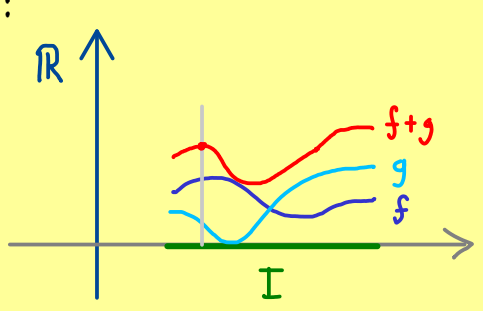
(b) Function space. Consider a set I and functions $f: I \longrightarrow \mathbb{R}$.

Then $\mathcal{F}(I) := \{ f: I \longrightarrow \mathbb{R} \}$ defines a real vector space:

- vector addition $f + g$ defined by:

$$(f+g)(x) := f(x) + g(x)$$

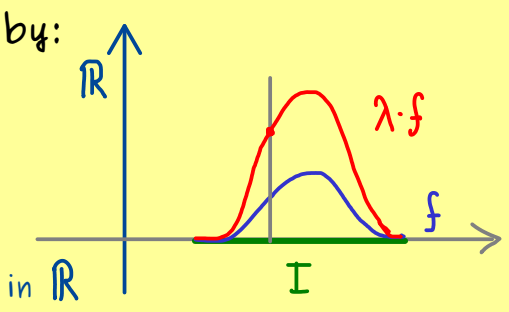
(+: addition in $\mathbb{R}$)



- scalar multiplication $\lambda \cdot f$ defined by:

$$(\lambda \cdot f)(x) := \lambda \cdot f(x)$$

(·: multiplication in $\mathbb{R}$)



↳ check 8 rules!

(c) Space of polynomials: $\mathcal{P}(\mathbb{R}) := \{ p: \mathbb{R} \to \mathbb{R} \text{ polynomial function} \}$

↳ $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$

$p_1 + p_2$, $\lambda \cdot p$ defined as before

$\Rightarrow$ real vector space

We see: $\mathcal{P}(\mathbb{R}) \subseteq \mathcal{F}(\mathbb{R})$

(⊆: linear subspace in $\mathcal{F}(\mathbb{R})$)