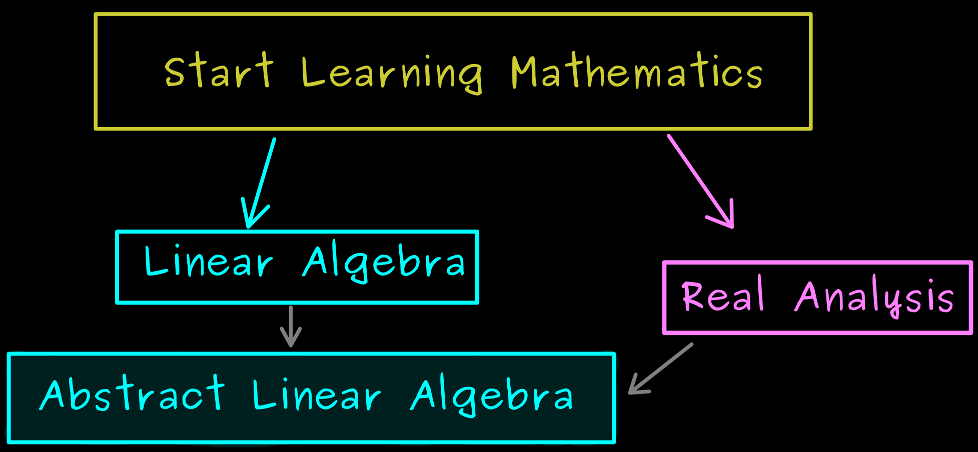
Abstract Linear Algebra – Part 1

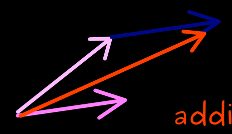
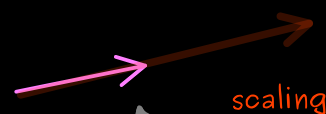
Prerequisites:



Content:

- general vector spaces
- general linear maps
- change of basis
- general inner products
- eigenvalue theory for linear maps

Definition: vector space $\boxed{V}$

$\left(\begin{array}{l} \mathbb{R}^n \\ \mathbb{C}^n \end{array} \right.$  adding vectors  scaling vectors $\Big)$

scalars from a field $\begin{cases} \mathbb{R} \\ \mathbb{C} \end{cases}$

Let $\mathbb{F}$ be a field (often $\mathbb{R}$ or $\mathbb{C}$).

A set $V \neq \emptyset$ together with two operations,

- vector addition $+ : V \times V \longrightarrow V$
- scalar multiplication $\cdot : \mathbb{F} \times V \longrightarrow V$

where the following eight rules are satisfied, is called an $\mathbb{F}$- vector space.

(a) $(V, +)$ is an abelian group:

(1) $u + (v + w) = (u + v) + w$ (associativity of $+$)

(2) $v + 0 = v$ with $0 \in V$ (neutral element)

(3) $v + (-v) = 0$ with $-v \in V$ (inverse elements)

(4) $v + w = w + v$ (commutativity of $+$)

(b) scalar multiplication is compatible:

(5) $\lambda \cdot (\mu \cdot v) = (\lambda \cdot \mu) \cdot v$

(6) $1 \cdot v = v$, $1 \in \mathbb{F}$ (multiplicative unit from the field)

(c) distributive laws:

(7) $\lambda \cdot (v + w) = \lambda \cdot v + \lambda \cdot w$

(8) $(\lambda + \mu) \cdot v = \lambda \cdot v + \mu \cdot v$ $\rightsquigarrow$ abstract vector space