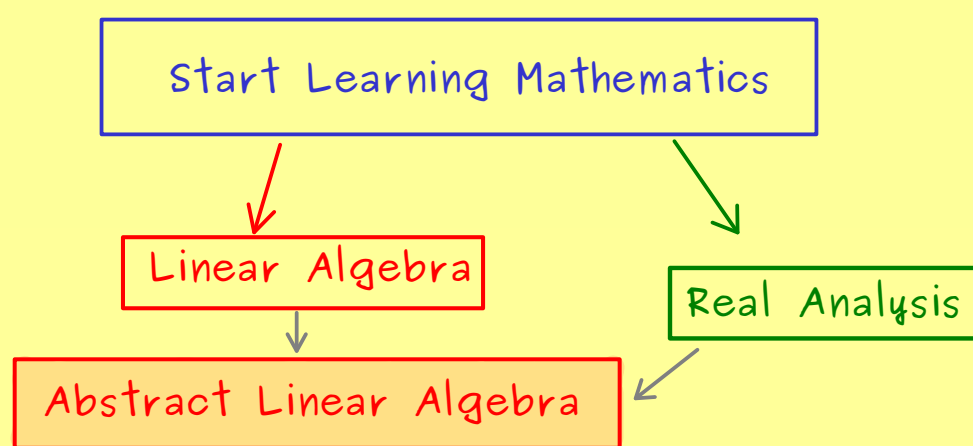




Abstract Linear Algebra - Part 1

Prerequisites:



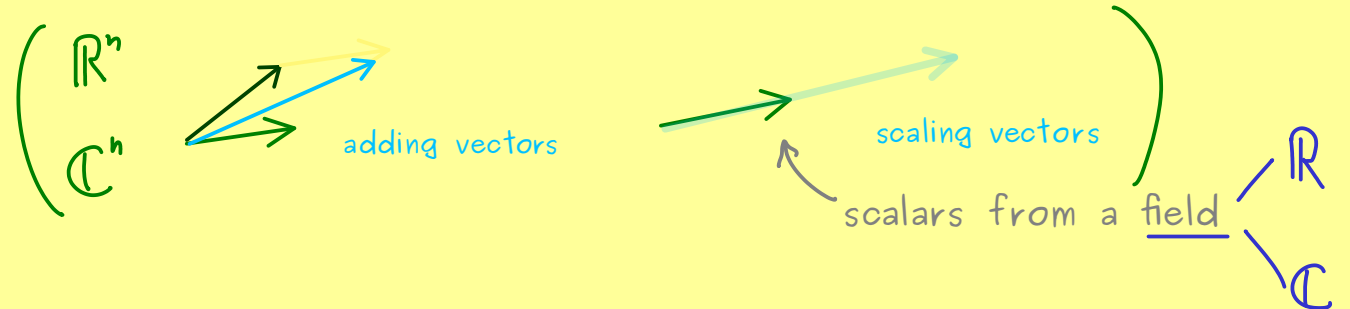
Content:

- general vector spaces
- general linear maps
- change of basis
- general inner products
- eigenvalue theory for linear maps

Definition:

vector space

V



Let \mathbb{F} be a field (often \mathbb{R} or \mathbb{C}).

A set $V \neq \emptyset$ together with two operations,

- vector addition $+$: $V \times V \rightarrow V$
- scalar multiplication \cdot : $\mathbb{F} \times V \rightarrow V$

where the following eight rules are satisfied, is called an \mathbb{F} -vector space.

(a) $(V, +)$ is an abelian group:

- (1) $u + (v + w) = (u + v) + w$ (associativity of $+$)
- (2) $v + 0 = v$ with $0 \in V$ (neutral element)
- (3) $v + (-v) = 0$ with $-v \in V$ (inverse elements)
- (4) $v + w = w + v$ (commutativity of $+$)

(b) scalar multiplication is compatible:

- (5) $\lambda \cdot (\mu \cdot v) = (\lambda \cdot \mu) \cdot v$
- (6) $1 \cdot v = v$, $1 \in \mathbb{F}$ (multiplicative unit from the field)

(c) distributive laws:

- (7) $\lambda \cdot (v + w) = \lambda \cdot v + \lambda \cdot w$
- (8) $(\lambda + \mu) \cdot v = \lambda \cdot v + \mu \cdot v$

\rightsquigarrow abstract vector space

