Complex Analysis – Part 14

$i^4 = \underbrace{i\cdot i\cdot i\cdot i}_{4\ \text{times}}$, $4^i = ?$

$\boxed{\log(x\cdot y) = \log(x) + \log(y)}$

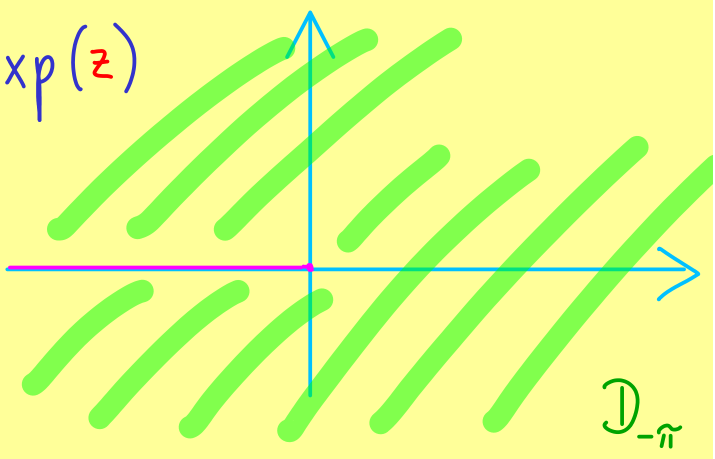
Power definition in $\mathbb{R}$: $a>0$, $m,n\in\mathbb{Z}\setminus\{0\}$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \exp\left(\log\left(\left(a^{\frac{1}{n}}\right)^m\right)\right) = \exp\left(m\cdot\log\left(a^{\frac{1}{n}}\right)\right) = \exp\left(\tfrac{m}{n}\cdot\log(a)\right)$$

$a>0$, $x\in\mathbb{R}$: $a^x := \exp(x\cdot\log(a))$

Power definition in $\mathbb{C}$: $a>0$, $z\in\mathbb{C}$: $a^z := \exp(z\cdot\log(a))$

for example: $e^z = \exp(z)$



for complex base? $a\in\mathbb{D}_{-\pi}$, $z\in\mathbb{C}$:

$$a^z := \exp(z\cdot\log(a))$$

principal value of the logarithm

principal value of the power

be careful in calculations: $a^{z_1}\cdot a^{z_2} = a^{z_1+z_2}$ ✓ $\left(a^{z_1}\right)^{z_2} \overset{\text{in general}}{\neq} a^{z_1\cdot z_2}$