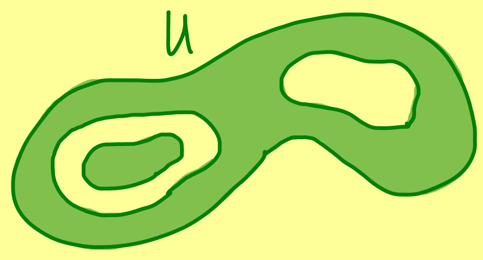
Complex Analysis – Part 4

Definition: $U \subseteq \mathbb{C}$ open. $f: U \to \mathbb{C}$ is called holomorphic (regular/ (complex) analytic/...) (on U)



if f is (complex) differentiable at every $z_0 \in U$.

If $U = \mathbb{C}$, the holomorphic function is called entire.

Properties: (a) f is holomorphic $\Rightarrow$ f is continuous

(b) $f, g : U \to \mathbb{C}$ holomorphic $\Rightarrow$ $f+g$, $f \cdot g$ holomorphic

(c) Sum rule, product rule, quotient rule and chain rule for derivatives hold.

Examples: (1) $f: \mathbb{C} \to \mathbb{C}$, $f(z) = a_m \cdot z^m + a_{m-1} z^{m-1} + \cdots + a_1 \cdot z^1 + a_0$ with $a_0, \dots, a_m \in \mathbb{C}$

A polynomial is an entire function.

$$f'(z) = m \cdot a_m \cdot z^{m-1} + (m-1) \cdot a_{m-1} z^{m-2} + \cdots + 2 \cdot a_2 \cdot z^1 + a_1$$

(2) $f: \mathbb{C} \setminus \{0\} \to \mathbb{C}$, $f(z) = \frac{1}{z}$ is holomorphic

(3) $f: \mathbb{C} \setminus \underbrace{S}_{=\{z \in \mathbb{C} \mid q(z) = 0\}} \to \mathbb{C}$, $f(z) = \frac{p(z)}{q(z)}$ is holomorphic

polynomial

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